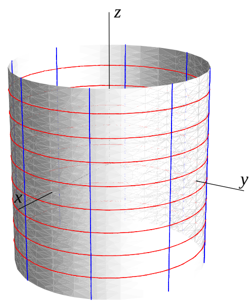
Parametric Surfaces

Last time, we learned that we could go from cylindrical coordinates (r, θ, z) or spherical coordinates (ρ, θ, ϕ) to Cartesian coordinates (x, y, z) using

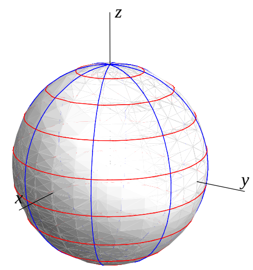
cylindrical	spherical
$x = r\cos\theta$	$x = \rho\sin\phi\cos\theta$
$y = r\sin\theta$	$y = \rho\sin\phi\sin\theta$
$z = z$	$z = \rho\cos\phi$

In the last problem we did in class, we looked at the cylinder $r = 5$ in cylindrical coordinates and saw that $\theta = k$ and $z = k$ (k a constant) formed a grid on the cylinder. Similarly, in spherical coordinates, we looked at the sphere $\rho = 5$ and saw that $\theta = k$ and $\phi = k$ formed a grid on the sphere.

cylindrical $r = 5$



spherical $\rho = 5$

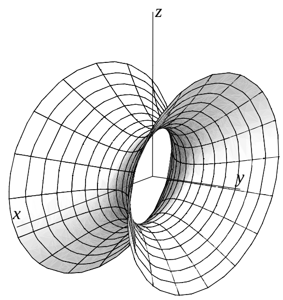


1. (a) Parameterize the elliptic paraboloid $z = x^2 + y^2 + 1$. Sketch the grid curves defined by your parameterization.

   (b) If we only want to parameterize the part of the elliptic paraboloid under the plane $z = 10$, what restrictions would you place on the parameters you used in (a)?

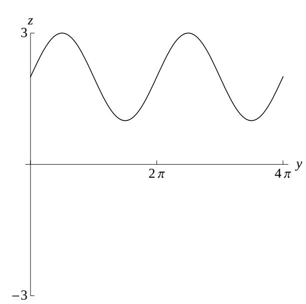
2. Parameterize the plane that contains the 3 points $P(1, 0, 1)$, $Q(2, -2, 2)$, and $R(3, 2, 4)$.

3. Parameterize the hyperboloid $x^2 - 4y^2 + z^2 = 1$.

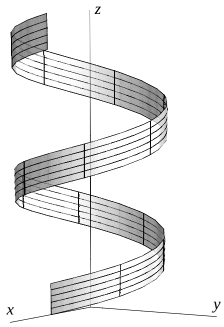


4. Parameterize the ellipsoid $9x^2 + 4y^2 + z^2 = 36$.

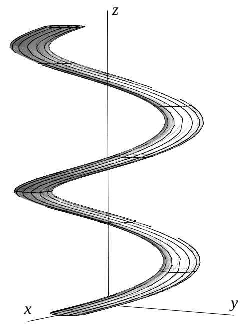
5. Consider the curve $z = 2 + \sin y$, $0 \leq y \leq 4\pi$ in the yz-plane. Let S be the surface obtained by rotating this curve about the y-axis. Find a parameterization of S.



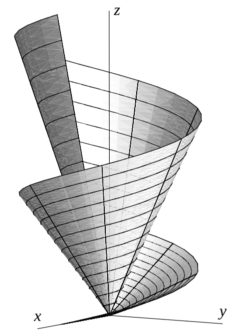
6. Here are three surfaces.



(I)



(II)



(III)

Match each function with the surface it parameterizes. Which curves are where u is constant and which curves are where v is constant?

(a) $\vec{r}(u,v) = \left\langle \dfrac{\cos u}{4} + \cos v, \dfrac{\sin u}{4} + \sin v, v \right\rangle$, $0 \le u \le 2\pi$, $0 \le v \le 4\pi$.

(b) $\vec{r}(u,v) = \left\langle \cos u, \sin u, u + \dfrac{v}{4} \right\rangle$, $0 \le u \le 4\pi$, $0 \le v \le 2\pi$.

(c) $\vec{r}(u,v) = \langle u\cos v, u\sin v, uv \rangle$, $0 \le u \le 2\pi$, $0 \le v \le 4\pi$.